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**Probability Theory and Applications (MA208)**
**Problem Sheet - 2**

**Finite Sample Spaces**

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1. The following group of persons is in a room: 5 men over 21, 4 men under 21, women over 21, and 3 women under 21. One person is chosen at random. The following vents are defined:

   $A = \{\text{the person is over 21}\}$; $B = \{\text{the person is under 21}\}$;

   $C = \{\text{the person is male}\}$; $D = \{\text{the person is female}\}$.

   Evaluate the following.

   (a) $P(B \cup D)$

   (b) $P(\overline{A} \cap \overline{C})$

2. Ten persons in a room are wearing badges marked 1 through 10. Three persons are chosen at random, and asked to leave the room simultaneously. Their badge number is noted.

   (a) What is the probability that the smallest badge number is 5?

   (b) What is the probability that the largest badge number is 5?

3. (a) Suppose that the three digits $1, 2$, and $3$ are written down in random order. What is the probability that at least one digit will occupy its proper place?

   (b) Same as (a) with the digits $1, 2, 3$, and $4$.

   (c) Same as (a) with the digits $1, 2, 3, \ldots, n$.

   (d) Discuss the answer to (c) if $n$ is large.

4. A shipment of 1500 washers contains 400 defective and 1100 nondefective items. Two-hundred washers are chosen at random (without replacement) and classified.

   (a) What is the probability that exactly 90 defective items are found?

   (b) What is the probability that at least 2 defective items are found?

5. Ten chips numbered 1 through 10 are mixed in a bowl. Two chips numbered $(X, Y)$ are drawn from one bowl, successively and without replacement. What is the probability that $X + Y = 10$?

6. A lot consists of 10 good articles, 4 with minor defects, and 2 with major defects. One article is chosen at random. Find the probability that:

   (a) it has no defects,

   (b) it has no major defects,

   (c) it is either good or has major defects.

7. A lot consists of 10 good articles, 4 with minor defects, and 2 with major defects. If two articles are chosen (without replacement), find the probability that:

(a) both are good,

(b) both have major defects,

(c) at least one is good,

(d) at most one is good,

(e) exactly one is good,

(f) neither has major defects,

(g) neither is good.

8. A product is assembled in three stages. At the first stage there are 5 assembly lines, at the second stage there are 4 assembly lines, and at the third stage there are 6 assembly lines. In how many different ways may the product be routed through the assembly process?

9. An inspector visits 6 different machines during the day. In order to prevent operators from knowing when he will inspect he varies the order of his visits. In how many ways may this be done?

10. A complex mechanism may fail at 15 stages. If it fails at 3 stages, in how many ways may this happen?

11. There are 12 ways in which a manufactured item can be a minor defective a 10 ways in which it can be a major defective. In how many ways can 1 minor and 1 major defective occur? 2 minor and 2 major defectives?

12. A mechanism may be set at any one of four positions, say $a, b, c$, and $d$. There are 8 such mechanisms which are inserted into a system.

    (a) In how many ways may this system be set?

    (b) Assume that these mechanisms are installed in some preassigned (linear) order.
    How many ways of setting the system are available if no two adjacent mechanisms in the same position?

    (c) How many ways are available if only positions $a$ and $b$ are used, and these are used equally often?

    (d) How many ways are available if only two different positions are used and one of these positions appears three times as often as the other?

13. Suppose that from $N$ objects we choose $n$ at random, with replacement. What the probability that no object is chosen more than once? (Suppose that $n < N$.)

14. From the letters $a, b, c, d, e$, and $f$ how many 4-letter code words may be form if,

    (a) no letter may be repeated?

    (b) any letter may be repeated any number of times?

15. Suppose that $\binom{99}{5} = a$ and $\binom{99}{4} = b$. Express $\binom{100}{95}$ in terms of $a$ and $b$. [Hint: Do not evaluate the above expressions to solve this problem.]

16. A box contains tags marked $1, 2, \ldots, n$. Two tags are chosen at random. Find the probability that the numbers on the tags will be consecutive integers if

    (a) the tags are chosen without replacement,

    (b) the tags are chosen with replacement.

17. How many subsets can be formed, containing at least one member, from set of 100 elements?

18. One integer is chosen at random from the numbers $1, 2, \ldots, 50$. What is the probability that the chosen number is divisible by 6 or by 8?

19. From 6 positive and 8 negative numbers, 4 numbers are chosen at random (without replacement) and multiplied. What is the probability that the product is positive number?

20. A certain chemical substance is made by mixing 5 separate liquids. It is proposed to pour one liquid into a tank, and then to add the other liquids in turn. All possible combinations must be tested to see which gives the best yield. How many tests must be performed?

21. A lot contains $n$ articles. If it is known that $r$ of the articles are defective and the articles are inspected in a random order, what is the probability that the $k$th articles $(k \geq r)$ inspected will be the last defective one in the lot?

22. $r$ numbers $(0 < r < 10)$ are chosen at random (without replacement) from the numbers $0, 1, 2, \ldots, 9$. What is the probability that no two are equal?

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